

## Summation Notes

**Ex 1.** Using the formula  $3k$ , substitute the numbers from 1-8 into the formula and add it up.

$$3(1)+3(2)+3(3)+3(4)+3(5)+3(6)+3(7)+3(8)$$

$$3+6+9+12+15+18+21+24$$

$$=108$$

**Ex 2.** Using the formula  $w^2 - 4$ , substitute the numbers from 1-5 into the formula then add them up.

$$(1^2 - 4) + (2^2 - 4) + (3^2 - 4) + (4^2 - 4) + (5^2 - 4) =$$

$$(1 - 4) + (4 - 4) + (9 - 4) + (16 - 4) + (25 - 4) =$$

$$-3 + 0 + 5 + 12 + 21 = 35$$

Do you think there is a "math way" to write what I was asking you to do in #1 and #2?

A series is an expression that indicates the SUM of terms of a sequence.

Summation notation uses  $\sum$  (sigma) as a shortcut to writing a series.

For example

Values of  $n$  from 1 to 5 are called the index  $\sum_{n=1}^5 2n$  Explicit formula for the general term of the sequence

substitute  $n=1$   $2(1) = 2$

substitute  $n=2$   $2(2) = 4$

substitute  $n=3$   $2(3) = 6$

substitute  $n=4$   $2(4) = 8$

substitute  $n=5$   $2(5) = 10$

Then, add all of the terms.

$$2 + 4 + 6 + 8 + 10 = 30$$

Find the sum of each finite series.

$$\begin{aligned}
 1. \sum_{j=1}^6 (j-2) &= (1-2) + (2-2) + (3-2) + (4-2) + (5-2) + (6-2) \\
 &= -1 + 0 + 1 + 2 + 3 + 4 \\
 &= 9
 \end{aligned}$$

$$\begin{aligned}
 2. \sum_{n=1}^3 3^n &= 3^1 + 3^2 + 3^3 \\
 &= 3 + 9 + 27 \\
 &= \boxed{39}
 \end{aligned}$$

Write each arithmetic series in summation notation.

3.  $7 + 9 + 11 + \dots + \boxed{21}$   $a_n$

arithmetic  $d=2$   
 $a_n = a_1 + d(n-1)$   
 $a_n = 7 + 2(n-1)$   
 $= 7 + 2n - 2$

explicit  
 Find  $a_n = 5 + 2n$  or  $2n + 5$

$\frac{21}{2} = \frac{5 + 2n}{2}$   
 $16 = 2n$   $n = 8$

First, we need an explicit formula from this sequence

Next, we need to know how many things to add up.

$$\sum_{n=1}^8 (2n+5)$$

4.  $(-3) + (-6) + (-9) + \dots + (-30)$

arithmetic  $d = -3$   
 $a_n = a_1 + d(n-1)$   
 simplify  $a_n = -3 + -3(n-1)$   
 $= -3 + (-3n) + 3$   
 $a_n = -3n$  ← explicit formula

Find  $a_n = -30 = -3n$   
 $\frac{-30}{-3} = \frac{-3n}{-3}$   
 $n = 10$

$$\sum_{n=1}^{10} (-3n)$$